QUESTION 2010

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following:

i) The value of $\log_3 27$ is

a) 5 ✓b) 3 c) 4 d) 2

ii) If $P(n)=n^2-n+41, \forall n \in \mathrm{N}$ then $P(n)$ is

a) an even number
b) a number divisible by 3
✓c) a prime number
d) a number divisible by 7

iii) If α and β are the roots of the equation $x^2+6x+2=0$, then the value of $\alpha^2+\beta^2$ is

✓a) 32 b) 30 c) 28 d) 2

iv) If A and B are any two sets then $A \cap (A \cup B)^c =$

✓a) ϕ b) B^c c) A^c d) B

v) If $A \propto B$, then

a) $A^3+B^2 \propto AB$ b) $A+B^2 \propto AB$ c) $A+B \propto A^2B$ ✓d) $A^2+B^2 \propto AB$

vi) The slope of the straight line that is parallel to line joining the points $(2,-1)$ and $(0,2)$ is

a) 1 b) $\frac{1}{3}$ c) $-\frac{1}{2}$ d) $\frac{1}{2}$

Correct answer is $-\frac{3}{2}$.

vii) The term, which is independent of x, in the expansion of $\left(x-\frac{3}{x}\right)^7$ is

a) 4 b) -3 c) 0 d) 3

No correct alternative is given. There is no term in the given expansion which is independent of x.

viii) The number of subsets of a set with n elements is

a) $2n$ ✓b) 2^n c) $\frac{n}{2}$ d) n^2

ix) The function $f(x)=x^2-2x+2$ is a

a) Even function b) Odd function c) Both (a) & (b) ✓d) Neither (a) nor (b)

x) π is a/an

a) natural number b) rational number ✓c) irrational number d) complex number

xi) The number of ways in which 4 letters can be ported in 5 letter boxes is

a) 4^5 ✓b) ${}^5P_4$ c) 5^4 d) ${}^5C_4$

xii) The coordinates of the centroid of the triangle whose vertices are $(2,0)$, $(1,-3)$, $(-3,3)$ is

a) $(2,1)$ ✓b) $(0,0)$ c) $(-1,3)$ d) $(2,4)$

Group – B
(Short Answer Type Questions)

2. If $x \propto y+z$, $y \propto z+x$ and $z \propto x+y$, then prove that $\frac{k}{k+1}+\frac{l}{l+1}+\frac{m}{m+1}=1$, where k, l, m are the constants of variation.

See Topic: RATIO, PROPORTION AND VARIATION, Short Answer Type Question No. 1.

3. Find the equation of the locus of a point such that the difference of its distances from the points $(5,0)$ and $(-5,0)$ is always 5 units.

See Topic: TWO DIMENSIONAL COORDINATE GEOMETRY, Short Answer Type Question No. 9.

4. Without using Venn Diagram prove that

$$A\cup(B\cap C)=(A\cup B)\cap(A\cup C).$$

See Topic: SETS, Short Answer Type Question No. 1.

5. Show that $7\log\left(\frac{10}{9}\right)-2\log\left(\frac{25}{24}\right)+3\log\left(\frac{81}{80}\right)=\log 2.$

See Topic: BASIC ALGEBRA, Short Answer Type Question No. 1.

6. In how many ways can the letters of the word "BALLOON" be arranged, so that two 'O's do not come together?

See Topic: PERMUTATIONS AND COMBINATIONS, Short Answer Type Question No. 5.

Group – C
(Long Answer Type Questions)

7. a) If α and β are the roots of the equation $2x^2-4x+1=0$, then form such an equation, whose roots are $\alpha^2+\beta$ and $\beta^2+\alpha$.

See Topic: THEORY OF QUADRATIC EQUATION, Long Answer Type Question No. 10.

b) Show that $\frac{1}{1+\log_x yz}+\frac{1}{1+\log_y zx}+\frac{1}{1+\log_z xy}=1$

See Topic: BASIC ALGEBRA, Long Answer Type Question No. 16.

c) Find the sum of series
$1.2+2.3+3.4+\ldots\ldots+$ upto n terms.

See Topic: SEQUENCES & SERIES, Long Answer Type Question No. 9.

8. a) Find the locus of the point, the ratio of whose distances from the line $x=2$ and from the point $(5,-1)$ is $3:2$.

See Topic: TWO DIMENSIONAL COORDINATE GEOMETRY, Long Answer Type Question No. 11.

b) If the coefficient of x^3 in the expansion of $\left(x^2+\frac{k}{x}\right)^6$ be 160, find the value of k.

See Topic: MATHEMATICAL INDUCTION & BINOMIAL THEOREM, Short Answer Type Question No. 4.

c) Find the equation of the circle through the points $(4,3)$ and $(-2,5)$ and having its centre on the line $2x-3y=4$.

See Topic: TWO DIMENSIONAL COORDINATE GEOMETRY, Long Answer Type Question No. 12.

9. a) What is the present value of Rs. 1,000 due in 2 years at 5% compound interest according as the interest is paid
   i) yearly
   ii) half-yearly.

See Topic: COMPOUND INTEREST AND ANNUITY, Long Answer Type Question No. 2.

b) Apply the principle of mathematical induction to prove,

$$\frac{1}{4.7}+\frac{1}{7.10}+\frac{1}{10.13}+\ldots\ldots+\frac{1}{(3n+1)(3n+4)}=\frac{n}{4(3n+4)}.$$

See Topic: MATHEMATICAL INDUCTION & BINOMIAL THEOREM, Long Answer Type Question No. 5.

c) Solve for $x: 2^{x+2}+2^{x-1}=9$.

See Topic: BASIC ALGEBRA, Long Answer Type Question No. 17.

10. a) Find the sum of the series $5+55+555+\ldots\ldots+$ upto n terms.

See Topic: SEQUENCES & SERIES, Long Answer Type Question No. 10.

b) Find the square root of $12-\sqrt{68+48\sqrt{2}}$.

See Topic: BASIC ALGEBRA, Long Answer Type Question No. 18.

c) Prove that the three points $(-2,-2)$, $(2,2)$ and $\left(-2\sqrt{3}, 2\sqrt{3}\right)$ are vertices of equilateral triangle. Find the area of the triangle.

See Topic: TWO DIMENSIONAL COORDINATE GEOMETRY, Long Answer Type Question No. 13.

11. a) In a class of 50 students, 15 read physics, 20 read chemistry, 20 read mathematics, 3 read physics & chemistry, 6 read chemistry & mathematics and 5 read physics & mathematics, 7 read none of the subject. How many students read all the subjects?

See Topic: SETS, Long Answer Type Question No. 9.

b) Find the total number of arrangements of the letters of the word "STATISTICS" when

i) there is no restriction
ii) the vowels remain together
iii) order of the vowels remain unchanged.

See Topic: PERMUTATIONS AND COMBINATIONS, Long Answer Type Question No. 2.